Observations on oblique shock waves in gaseous detonations

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An account is given of photographic and pressure observations made on the oblique shock waves occurring in the wake of self-sustaining detonation waves in hydrogen-oxygen mixtures initially at atmospheric pressure. Four explosion tubes were employed, of which three are of circular cross-section with internal diameters of 10, 5 and 1·6 cm and the fourth is a square-section tube of side 1·5 in.

On the assumption that the oblique shocks are sufficiently weak to be regarded as Mach waves, the flow Mach number relative to the detonation front is determined; these are found to be substantially higher than the values predicted by ideal one-dimensional theory. The measured flow Mach numbers in the rarefaction are then used to calculate the pressure distribution in this region on the basis of the supersonic nozzle model due to Fay (1959, 1962). The predictions of this model are found to disagree with the observed static pressure profiles. Moreover, the pressure following the initial peak persists at a higher value than the theoretical for distances of the order of 5–10 cm behind the front. This phenomenon implies that the wall boundary-layer pressure remains higher than the C–J value and it is suggested that the pressure difference across the boundary layer can account for the formation of the oblique waves.

The supersonic features of the flow can be accounted for by the turbulent-structure hypothesis of White (1961). Some validation of this hypothesis is provided here by the observation of the absence of the oblique shocks in over-driven detonation waves caused by the diminished effects of turbulence. This observation is consistent with the view that the oblique shocks are generated by the pressure difference across the boundary layer near the front as this difference would also be diminished in an over-driven wave.

1. Introduction

The occurrence of weak compression pulses, of regular periodicity, in the wake of the detonation front in hydrogen-oxygen mixtures confined in tubes of various diameters, has previously been reported from this Laboratory (Edwards, Williams & Breeze 1959; Edwards & Jones 1960). On wave-speed schlieren photographs these pulses appear as uniformly spaced bands travelling with the same velocity as the detonation front and their pressure amplitude, although weak on static recordings, is quite pronounced in the reflexion-pressure records.

At first, the origin of these compressions was incorrectly ascribed to disturbances occurring downstream and arising from continuing chemical reaction in the rarefaction wave in accordance with the mechanism proposed by Brinkley & Richardson (1953) to explain the build up of an ideal Chapman–Jouguet (C–J) wave. In subsequent work, however, streak photographs taken through narrow vertical windows placed perpendicular to the detonation tube axis, have revealed that the compressions form a part of a wave pattern of oblique shocks originating at the detonation front and which is stationary with respect to the front. Such waves have been observed by Fay & Opel (1958) in $C_2H_2+\frac{3}{2}O_2$ at an initial pressure of 0·25 atm. These authors suggest that the waves are Mach waves which originate from boundary-layer effects near the reaction zone. An analysis of the flow divergence in the detonation front arising from the wall boundary-layer displacement effect in the subsonic reaction zone has been successfully applied by Fay (1959) to the quantitative interpretation of the phenomenon of wave velocity deficits in terms of tube diameter and width of the reaction zone. By the same reasoning, if further isentropic expansion occurs at the sonic equilibrium C–J point, supersonic flow will result and by analogy with supersonic nozzle flow, oblique shock waves are established at the interaction point formed by the C–J surface and the boundary layer. Successive reflexions of the waves at the tube wall will then give rise to the observed wave pattern. The Mach lines recorded by Fay & Opel (1958) in a 2 in. square tube correspond to flow Mach numbers in the range 1·10–1·15 depending on the particular mixture composition (Fay 1962). Moreover, values of the ratios of densities for the expanded flow to that obtaining at the C–J surface for these Mach numbers are found to offer a possible explanation for the ambiguity in the density measurements of Duff, Knight & Rink (1958), assuming that the equilibrium and not the frozen value of sound speed is appropriate for the calculation.

An alternative explanation for the supersonic flow observed in the rear of the reaction zone and the discrepancy between measured densities and those computed on the assumption of equilibrium sound speed at the C–J plane, is offered by the discovery by White (1961) of the turbulent nature of the flow behind the reaction zone of a self-sustaining detonation wave. The consequences of turbulence (in the region following the reaction zone) are shown to be a lower pressure and density than calculated according to the C–J hypothesis and a mass flow with respect to the wave front which is supersonic.

It may be that the effect of both turbulent structure and two-dimensional flow engendered by the wall boundary layer contribute to the observed departures from plane idealized flow. In the light of existing experimental evidence, however, it is difficult to assess their relative importance. From a phenomenological standpoint both approaches have plausible features but their inherent weakness, in their present state, is their exclusion of spin, a phenomenon which appears to be inseparable from self-sustaining detonation waves propagated in tubes.

In the present paper photographic records of oblique waves in detonation waves in hydrogen-oxygen mixtures are described together with static pressure records, whose profiles, it is suggested, are of relevance in the interpretation of the oblique waves.

2. Experimental apparatus

The detonation tubes employed for pressure measurement and for wave-speed photographs taken through windows parallel to the tube axis are described elsewhere (Edwards *et al.* 1959). Briefly, they are of three diameters, 1·6, 5 and 10 cm and of circular cross-section. Each tube consists of three sections, a channel of length 4 m, a 'driver' or 'chamber' of length 1 m and an optical section containing glass windows 10 cm long. Hydrogen and oxygen, obtained from commercial cylinders, are premixed, in various proportions, in large cylinders before they are admitted into the previously evacuated channel. The initial pressure of the channel mixtures in all the experiments referred to in this work is atmospheric.

Detonation is usually initiated by means of a stoichiometric acetylene-oxygen mixture, at atmospheric pressure, contained in the driver and which is initially isolated from the channel by a 'Melinex' diaphragm. The driver gas is fired by a powerful spark or copper acetylide matchhead. Alternatively, initiation can also be achieved by transmitting a strong shock into the channel mixture by gradually pressurizing the driver with cold hydrogen until the diaphragm ruptures, as in a conventional shock tube.

Velocity measurements are made by means of ionization probes placed at intervals along the tube wall. These consist of slightly modified 10 mm spark plugs used in model engines. Average values of velocities over two 20 cm lengths are obtained from the readings of two microsecond chronometers.

As a suitable repetitive light-flash source was not available, time-distance schlieren displays were obtained on a rotating-mirror camera by photographing the waves through a pair of vertical windows, stopped down to a width of 1 mm. In order to examine conditions across the entire diameter of the flow a $1\frac{1}{2}$ in. square-section tube of stainless steel was instrumented for this purpose; in other respects it is similar to the tubes described above. By inserting a prism between the knife-edge and the camera the schlieren image could be rotated through 90° so as to lie accurately perpendicular to its direction of motion on the photographic film. The schlieren system is a twin-mirror type employing 12 in. diameter, 8 ft. focal-length mirrors. Image speeds of up to 0·7 mm/μsec can be obtained with the camera and this is determined at the instant of firing to an accuracy of 0·1 %.

The pressure gauges employed are of the pressure-bar type described by Edwards (1958). They are of two diameters, $\frac{1}{4}$ and $\frac{1}{2}$ in., and have rise-times of 3 and 6 μsec respectively. Calibration of the gauges is achieved by means of low-Mach-number waves in air generated in a 2 in. diameter shock tube. When the gauges are set in the static position the overshoot on loading is less than 5 %.

3. The photographic records

Typical photographs obtained in the 1·5 in. square-section tube by the method outlined above are shown in figure 1 (*a*) and (*b*), plate 1, and a diagram of the main features of record (*b*) is drawn in figure 1 (*c*). The mixture is $H_2 + O_2$ at an initial pressure of 1 atm. Shock waves generated at the interaction of the detonation front and the tube wall are clearly seen, together with their reflexion downstream, at the tube wall. When windows set parallel to the tube axis are used in the photography, the conventional streak picture is obtained in which only the points of intersection of the oblique waves are recorded. Such a photograph is shown in figure 2(*a*), plate 2, for a detonation wave in $3H_2+O_2$ at atmospheric initial pressure in the 10 cm diameter tube. It is evident from this photograph that the shock-wave pattern is stationary with respect to the detonation front. Occasionally the bands appear as double lines, as in figure 2(*a*), owing to the asymmetry of the pattern about the tube axis.

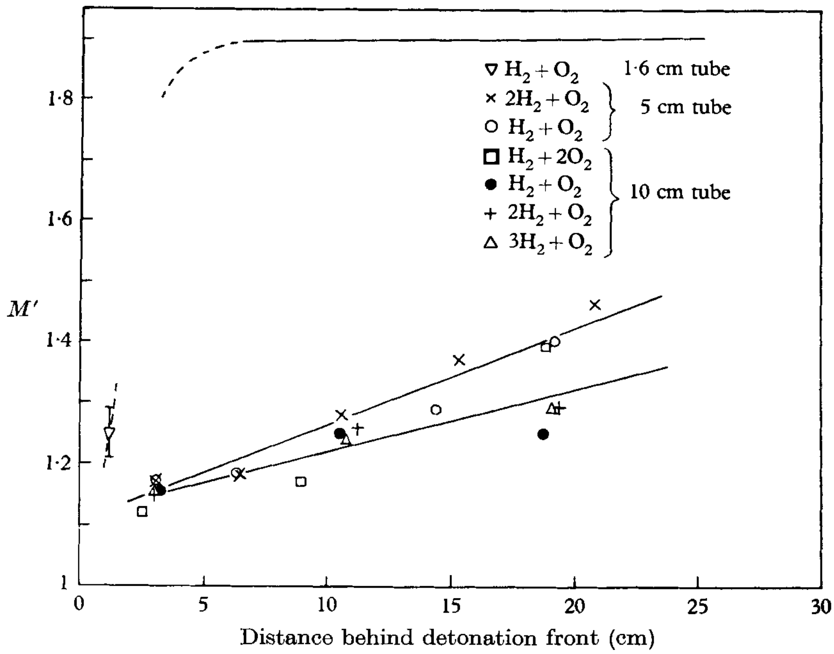

FIGURE 3. Observed variation of flow Mach number relative to the detonation front, M', with distance behind the front.

The criss-cross pattern observed behind the front in figure 1 is interpreted as a transverse spin of high modal number. As this photograph is a time-distance display the bands do not lie parallel to the time axis: the transverse wave is progressive and consequently the pressure and density patterns rotate about the tube axis. Inspection of the photographs shows that approximately equal numbers of bands travel upwards as downwards, which implies the existence of equal amounts of right- and left-handed spin.

In some photographs the rapid variation in the slope, α, of the shock wave originating at the detonation front is clearly discernible. At regions close to the reaction zone the acceleration of the gas would be expected to be large. Unfortunately, the quality of the photographs does not permit accurate measurement of this variation to be made. Consequently, only the average slopes, β, of these shock waves has been determined. If M' denotes the Mach number of the flow with respect to the detonation front and the oblique shocks are assumed to be sufficiently weak to be regarded as Mach waves, then

$$M' = \frac{u_1 - u_2}{a_2} = \frac{1}{\sin\beta},$$

where u_1, u_2 and a_2 are the detonation velocity, mass and sound velocities at the C–J plane, respectively.

For each detonation tube the measured values of M' are found to increase with distance behind the detonation front. The time over which the records of the oblique waves are found to be measurable is $\simeq 100\,\mu$sec, which corresponds to a distance $\simeq 20$ cm. This variation in flow Mach number is, in general, not very regular and in the 1·6 cm tube it is difficult to determine with accuracy. In figure 3 a plot is shown of the values of M' against distance behind the front, for each tube diameter, and several gas mixtures. The values of M' summarized in the last column of table 1, for all the mixtures studied in the four tubes, are the averaged values obtained from the two oblique compressions originating at the detonation front.

Tube diameter	Mixture	Observed velocity deficit, $\Delta u_1/u_1$ (%)	Calculated M' from equations (2) and (3)	Observed M' (at distance $\simeq 3$ cm behind front)
10 cm	$3H_2+O_2$	1·3	1·16	1·16
	$2H_2+O_2$	1	1·15	1·16
	H_2+O_2	0·6	1·12	1·16
	H_2+2O_2	1·6	1·19	1·12
5 cm	$2H_2+O_2$	0·2	1·08	1·17
	H_2+O_2	0·5	1·10	1·17
1·6 cm	H_2+O_2	0·8	1·13	1·25
1·5-in. square-section	$4H_2+O_2$	—	—	1·13
	$3H_2+O_2$	—	—	1·19
	$2H_2+O_2$	—	—	1·19
	H_2+O_2	—	—	1·18

TABLE 1

One particular feature of these records, which is invariably found for all mixtures in the composition range $4H_2+O_2$ to H_2+2O_2, is puzzling. This is the appearance of an expansion wave at one wall of the tube only and originating at a point a little downstream, $\sim 5\,\mu$sec, of the detonation front. Further photographs of improved quality are required to determine whether this phenomenon has real significance or is merely due to some peculiarity in the geometry of the optical section.

4. The pressure records

A detailed account of our pressure measurements has been given previously (Edwards *et al.* 1959; Edwards, Williams & Price 1962). For our present purposes it suffices to give the static pressure profiles for one hydrogen-oxygen mixture in each of the three circular-section detonation tubes. These are shown in

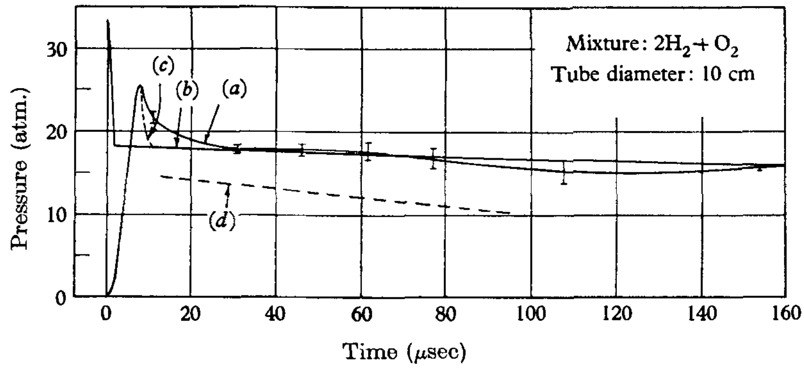

FIGURE 4. Curve (*a*) observed static pressure-time distribution for detonation wave in $2H_2+O_2$ in the 10 cm diameter tube; (*b*) calculated profile for ideal wave with $\gamma = 1{\cdot}2$; (*c*) expected gauge response to ideal wave; (*d*) calculated pressures assuming nozzle model and observed values of M' (figure 3).

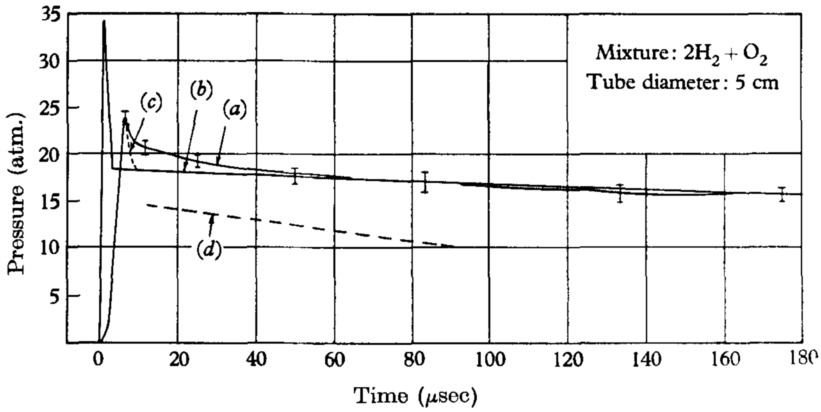

FIGURE 5. Curve (*a*) observed static pressure-time distribution for detonation wave in $2H_2+O_2$ in the 5 cm diameter tube. Curves (*b*), (*c*) and (*d*) as for figure 4.

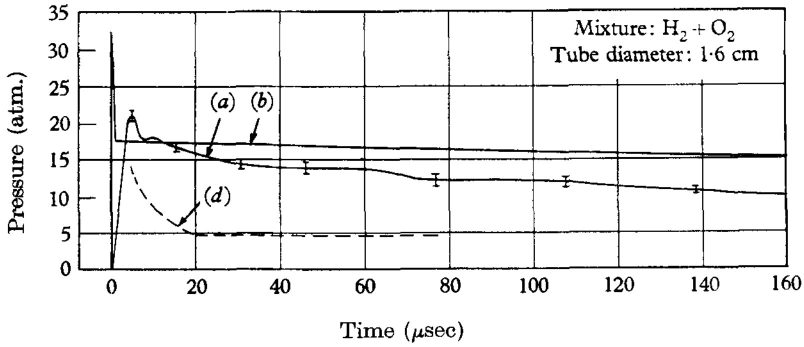

FIGURE 6. Curve (*a*) observed static pressure-time distribution for detonation wave in H_2+O_2 in the 1·6 cm diameter tube. Curves (*b*), (*c*) and (*d*) as for figure 4.

figures 4–6; the standard deviations of replicate measurements are indicated. Also drawn on the same diagrams are the calculated pressure variation in the rarefaction wave assuming idealized one-dimensional flow with constant $\gamma = 1{\cdot}2$. Kistiakowsky & Kydd (1955) have shown that neglect of dissociation effects in the computation does not lead to significant errors in the density and consequently in the pressure. The representation of the shape and width of the reaction zone is schematic. Naturally, the limitations of the high-frequency response of the gauge results in a very imperfect recording of the pressure variation in the reaction zone of waves propagating in mixtures at atmospheric initial pressure. Nevertheless, a partial response to the von Neumann peak is obtained

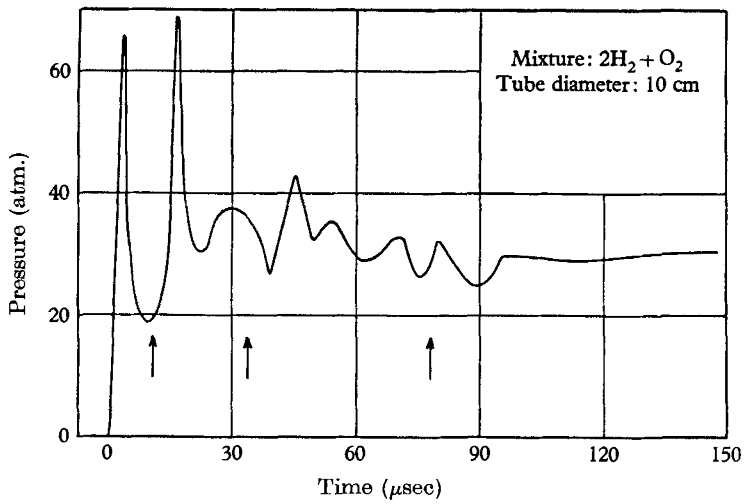

FIGURE 7. Pressure-time recording of the reflexion pressure for a detonation wave in $2H_2 + O_2$ in the 10 cm diameter tube. Arrows indicate the oblique shock-wave interactions measured from schlieren photographs.

whose shape, but not magnitude, can be predicted from pressure-gauge characteristics. In particular, the observed peak would be expected to be symmetrical, i.e. the decay-time from the peak value should be comparable to the rise-time as indicated in the diagrams of figures 4–6.

It can be seen that in the case of the 10 and 5 cm diameter tubes the pressure decays at a slower rate than predicted. In consequence it is concluded that the pressure in the wall boundary layer persists at a higher value than that in the interior of the tube for distances of the order of 5–10 cm downstream. This feature of the pressure records is observed (in these two tubes) for all mixtures apart from those near the limits of detonability which spin in a low-frequency mode. There is some evidence of the same effect in the 1·6 cm diameter tube, but is considerably less pronounced than in the larger tubes. Indeed, the behaviour observed in this tube differs markedly from that in the 10 and 5 cm tubes with regard to both the rapid decay of pressure in the rarefaction wave and the appreciably higher flow Mach number already noted in figure 3.

Two records are shown in figures 7 and 8 which were obtained with a $\frac{1}{4}$ in. diameter pressure gauge set in the end plate of the tube to measure the pressure arising from the normal reflexion of a detonation wave. These are of stoichiometric hydrogen-oxygen mixtures in the 10 and 1·6 cm tubes respectively. The peaks arising from the reflexion of the oblique waves are clearly identifiable from the measured positions derived from the corresponding photographic records, as indicated by the arrows. In the reflexion position the gauge is free of the effects of wall boundary layer and, therefore, the records provide confirmation of the expected behaviour of the gauge to a narrow pressure pulse, mentioned above. It is seen, particularly in the case of the 10 cm tube record, that the initial peak, representing the response of the gauge to the reflected von Neumann spike, is symmetrical. The decay of the pressure following this peak, and before the arrival of the second pressure peak, to a value of 18·5 atm., which is well below the calculated value of 44·2 atm. for the reflexion of the C–J plane, is most probably due to shock-bifurcation effects. Mark (1958) has demon-

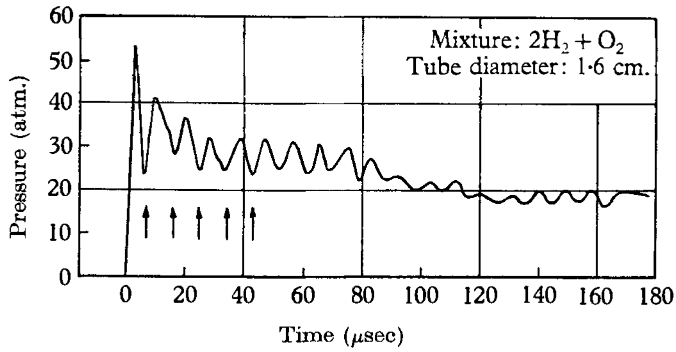

FIGURE 8. Pressure-time recording of the reflexion pressure for a detonation wave in $2H_2 + O_2$ in the 1·6 cm-diameter tube. Arrows indicate the oblique shock-wave interactions measured from schlieren photographs.

strated that, for polyatomic gases, a result of the interaction between a reflected shock wave and the boundary layer behind the incident wave is, initially, to cause a deceleration of the reflected wave as it moves away from the reflecting wall. The reflected wave does not remain plane and a fall in pressure at the wall ensues. Furthermore, as the value of the ratio of specific heats of the gas, γ, decreases, the effects of bifurcation increase. It follows that in detonation waves in which a low value of γ ($\simeq$ 1·2) obtains, conditions are particularly favourable for non-ideal reflexion.

5. Discussion

In order to avoid ambiguity in the discussion of the above experimental results it is necessary to define the region of flow which is being considered, viz. the flow in the rear of the steady reaction zone. The flow in the non-steady expansion wave, in the absence of wall perturbations or structural departures from ideality, is supersonic with respect to the detonation front. It is, therefore, necessary to inquire to what extent the Mach number of the flow calculated from the oblique shocks deviates from that expected from the predictions of the theory of plane ideal flow. If we use the analysis of Doring (1944) and Taylor

(1950) and assume a constant value of $\gamma = 1{\cdot}2$, we find the variation in Mach number for the flow in a tube in which the wave has travelled 5 m from the plane of initiation to be as shown, for the case of $H_2 + O_2$, in figure 9. In this diagram the width of the reaction zone is exaggerated and the variation of Mach number within it is arbitrarily indicated. At a distance of 20 cm downstream of the C–J plane the calculated value for M' in $H_2 + O_2$ is 1·05, and this value varies little with mixture composition, which is significantly different from the observed values at this point given in figure 3; these are 1·89, 1·42, 1·32 for the 1·6, 5 and 10 cm diameter tubes respectively. If the hypothesis of nozzle flow proposed by Fay & Opel (1958) is valid then the pressure variation in the isentropic flow of the expansion wave should be predictable from the Mach number (see Fay

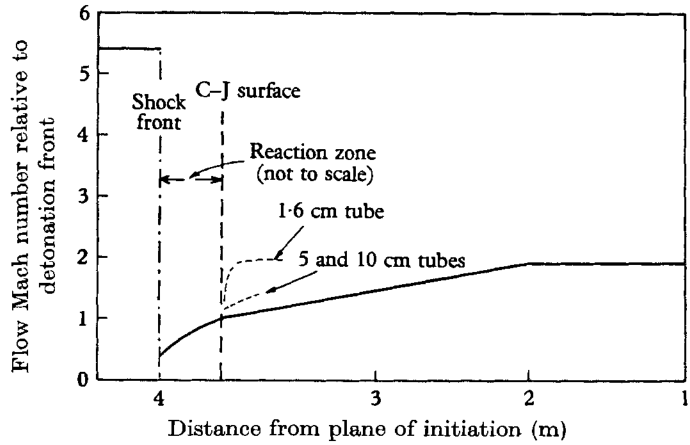

FIGURE 9. Graph of flow Mach number relative to the detonation front against distance from the plane of initiation for $H_2 + O_2$. Detonation is assumed to occur instantaneously at the closed end of a tube 5 m in length. Dotted curves show the observed Mach number in the 1·6, 5 and 10 cm diameter tubes.

1962). As the flow becomes sonic at the C–J plane the C–J pressure, $p_{\mathrm{C-J}}$, will correspond to the nozzle throat pressure and the pressure, $p_{M'}$, at any point downstream, where the flow Mach number is M', will be given by

$$\frac{p_{\mathrm{C-J}}}{p_{M'}} = \left[\frac{2}{\gamma+1}\left(1+\frac{\gamma+1}{2}M'^2\right)\right]^{\gamma/(\gamma-1)}. \tag{1}$$

Again, using a constant value for γ of 1·2, which is a reasonable approximation, values of $p_{\mathrm{C-J}}/p_{M'}$ have been computed for the various observed values of M'. The resulting pressure profiles for a duration $\simeq 100\,\mu$sec are shown in figures 4–6 for comparison with the experimental static-pressure profile and the calculated pressure variation in the expansion wave based on ideal theory. If the agreement between the measured and ideal pressure profiles is not fortuitous, and there appears to be no evidence to suggest this, then the results do not lend support to the nozzle flow model as presently interpreted. This conclusion would appear to be at variance with the achievement of the nozzle model, within the steady reaction zone, in rationalizing the observations on velocity deficits in finite-diameter tubes. It is, therefore, of relevance to attempt an examination of the velocity deficits observed in the present detonation tubes in terms of the nozzle model. Fay (1959) shows that the fractional decrease in wave velocity, $\Delta u_1/u_1$, is related to the fractional increase in cross-sectional area of the tubes at the C–J plane, ζ, that arises on account of the viscous boundary layer in the subsonic reaction zone, by the expression

$$\Delta u_1/u_1 = 0{\cdot}53\epsilon\zeta \quad (\zeta \ll 1,\ 1 < \epsilon < 2), \tag{2}$$

for stoichiometric hydrogen-oxygen mixtures. The numerical coefficient in this equation is expected to be approximately the same for all detonating mixtures. On the assumption, which is implicit in Fay's argument, that the same stream-tube area-increase is operative behind, and in the immediately vicinity of, the C–J plane then the flow Mach number in this region of isentropic expansion can be used to predict values of ζ and hence of $\Delta u_1/u_1$ by equation (2). ζ and M' are related by the nozzle expression

$$\zeta = \frac{1}{M'}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M'^2\right)\right]^{[\gamma+1]/[2(\gamma-1)]} - 1. \tag{3}$$

These calculations have been carried out for our measurements and are given in table 1. In view of the uncertainty in the numerical factor of equation (2), together with the fact that the measured flow Mach number M' is an averaged value derived over a distance of approximately 3 cm behind the front, the agreement between the two values in the case of the 10 and 5 cm tubes is reasonably good. However, the success of the nozzle theory in predicting the deviations of wave velocity from the values appropriate to infinite diameter tubes and its failure to establish the observed pressure distribution is not necessarily an inconsistency but serves to emphasize the relative insensitivity of detonation wave velocity to perturbation effects.

Another mechanism whereby the oblique shock waves arise is suggested by the static pressure profiles. Thus the records of figures 4–6 show that the pressure in the 10 and 5 cm tubes, and to a less marked extent in the 1·6 cm tube, does not attain the theoretical value in the time expected from the known gauge behaviour. This implies that the rate of chemical reaction is decreased in the boundary layer and as a result the pressure there is maintained at a higher value than in the free stream. In the supersonic region immediately behind the C–J plane a difference of pressure would give rise to a shock wave inclined to the flow in the manner shown in figure 10. After a time $\simeq$ 20–40 μsec the pressure in the boundary layer assumes the same value as the free-stream pressure.

If this explanation of the cause of the oblique waves is accepted then an answer must be sought for the high values found for the flow Mach numbers in the rarefaction wave. It would seem that elucidation of this phenomenon is offered by the turbulent structure hypothesis of White (1961). Having established experimentally that the flow in and behind the reaction zone is turbulent this author shows that when turbulent terms are added to the conservation equations the C–J state no longer exists and a self-sustaining detonation wave lies on the weak branch of the Rankine–Hugoniot curve. And, in particular, the flow in the state following the detonation front is supersonic with respect to the front. Another consequence of turbulence in a self-sustaining detonation wave is to decrease the pressure below its C–J value. In support of this White (1961) has shown that the pressures measured behind detonation fronts is $2H_2 + O_2 + 2CO$ at an initial pressure of 1 atm. give substantially lower values than calculated on the C–J hypothesis. Our present measurements in the 10 and 5 cm tubes do not confirm this deduction but the low pressures observed in the 1·6 cm tube

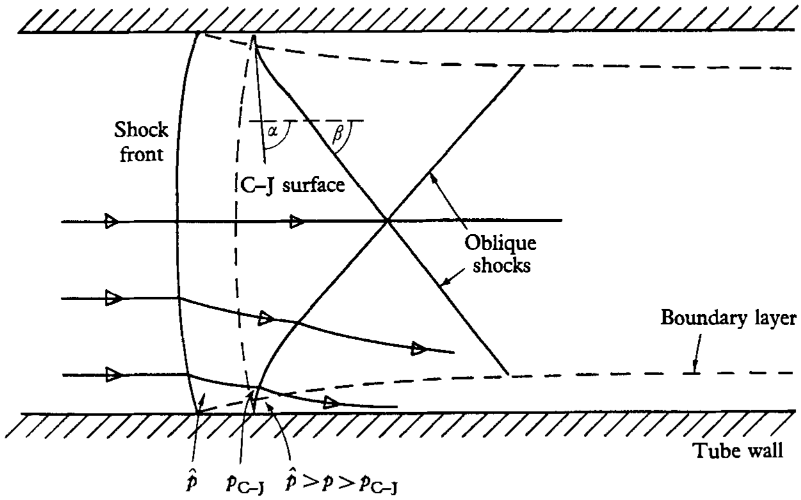

FIGURE 10. Sketch of two-dimensional flow behind a detonation wave showing the origin of the oblique shocks and divergence of the streamlines. $\hat{p}$ and p_{C-J} denote the shock and C–J pressures respectively.

(figure 6) may be attributable to this cause. However, until a quantitative evaluation of the expected decrease in pressure arising as a result of turbulence is available it would be premature to draw a definite conclusion.

A further point of significance arises when over-driven detonation waves are examined. White (1961) shows that in such waves the effects of turbulence are diminished and the densities and pressures found experimentally agree closely with theoretical predictions. Consequently, this author argues that as the boundary layer and curvature of the front are not greatly changed in comparison with a self-sustaining detonation, the agreement invalidates the nozzle flow model. It follows also that in an over-driven wave no oblique shocks would be found. In a preliminary investigation of over-driven waves our observations confirm this. The disappearance of the oblique waves in this case does not in any way contradict the proposed mechanism of their generation. For an over-driven wave the pressure gradient of the reaction zone is not as pronounced as in a self-sustaining detonation. Consequently, in the neighbourhood of the reaction zone, the pressure difference existing between inside and outside of the boundary layer would be diminished. It follows that when the driving wave is sufficiently strong the oblique shocks are not formed owing to the near equality of these pressures.

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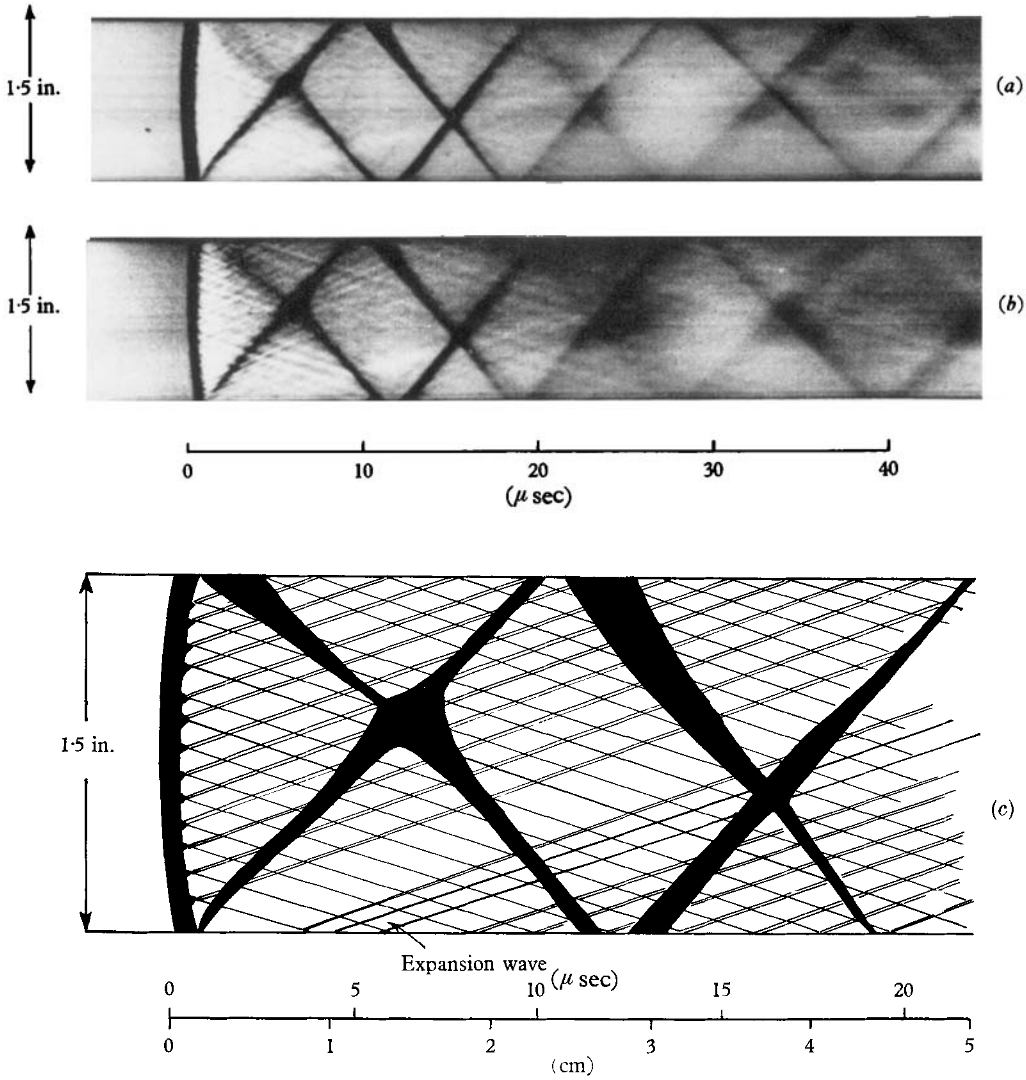

FIGURE 1. Schlieren photographs of detonation waves in 1·5 in. square-section tube for (*a*) $2H_2 + O_2$ and (*b*) $H_2 + O_2$ at an initial pressure of 1 atm. (*c*) Diagram showing the main features of photograph of figure 1 (*b*) for $H_2 + O_2$.

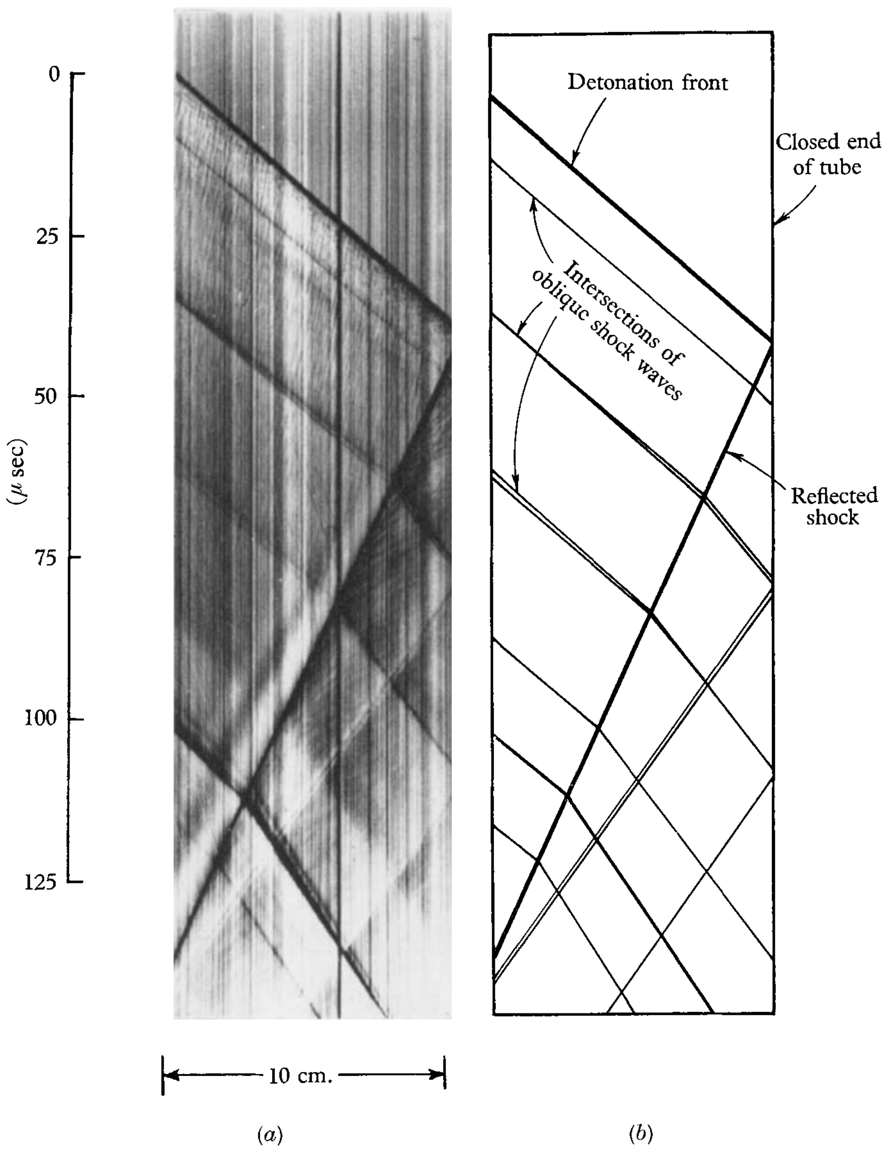

FIGURE 2. (*a*) Streak schlieren photograph of detonation wave in $3H_2 + O_2$ in 10 cm-diameter tube, (*b*) explanatory diagram of figure 2(*a*).